Urban Transit Network Design Problems: A Review of Population-based Metaheuristics

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Abstract – The urban transit network design problem (UTNDP) involves the development of a transit route set and associated schedules for an urban public transit system. The design of efficient public transit systems is widely considered as a viable option for the economic, social, and physical structure of an urban setting. This paper reviews four well-known population-based metaheuristics that have been employed and deemed potentially viable for tackling the UTNDP. The aim is to give a thorough review of the algorithms and identify the gaps for future research directions.

Keywords: Metaheuristics, public transit systems, route network, urban transit network design problem, urban transit routing problem, urban transit scheduling problem

Introduction

The urban transit network design problem (UTNDP) involves the development of a set of routes and schedules on a given road network, adhering to practical constraints for an urban public transportation system such as bus, train and tram. The public transportation system has been largely regarded as a viable option of mitigating air pollution, noise level, traffic congestion and accidents, energy consumption, and improving mobility. The social, economic, and physical structure of urban areas can be enhanced through the designing of an operationally and economically efficient public transportation system particularly, in developing and emerging countries (Fan & Machemehl, 2006). In addition, some of the attributes that can make public transportation system attractive include among others; capacity of route, frequency of service, service coverage, service reliability, comfort and service quality (Sinha, 2003; Vuchic, 2005). More so, a number of well-known indicators considered by numerous researchers to assess the quality of a transit route network as mentioned in Zhao & Zeng (2007) include service coverage, passenger cost, service quality, and operator cost.

A route network is constructed on a given road network at the transit network design phase of the UTNDP, where a sequence of adjacent stations and associated links of the road network constitute an individual route. Based on Desaulniers & Hickman (2007), this function is undertaken in the course of long-term planning, usually regulated by the authorities. Levinson (1992) noted that passenger flows should serve as a guide in transit route layout design, where routes are constructed to provide direct or indirect link between stations. In a formal sense, a route with more than two transfers between the origin and the destination is commonly unacceptable. Therefore, a transit network on which the majority of passengers who could not travel without transfer should be restructured (Zhao, Xu, & Jiang, 2015).

In the most general sense, the interest of the transit users and the operators is accounted for by the configuration of the transit route and the corresponding frequency. Hence, the higher the number of routes as well as high service frequencies results to a better service coverage provided to the transit users, but operators cost may tend to be higher thereby making their operation unprofitable. In such scenario, there is need to establish a suitable trade-off level through the evaluation of various alternative transit systems designs (Mauttone & Urquhart, 2009). Consequently, most of the studies in the literature resort to multiobjective formulation of the UTNDP. In 1998, Van Nes, Hamerslag, and Immers (1988) classified models for solving the UTNDP into the following perspectives: (1) analytical models for parameter configuration of the public transit system; (2) models determining the street segments (links) to be used for public transit route construction; (3) models for constructing routes only; (4) models assigning frequencies to a set of routes; (5) bi-level models for generating routes and assigning frequencies; and (6) models for simultaneously determining routes and frequencies.

Recently, several interesting reviews have been made available in the area of transportation planning. Guihaire and Hao (2008) focused on a review of the important strategic and tactical steps of transportation planning. Kepaptsoglo & Karlaftis (2009) summarised UTNDP research based on the design objectives, operating environment parameters and solution approaches. Farahani, Miandoabchi, Szeto, and Rashidi (2013) focused both on the road network design problem and the UTNDP by concentrating on the definitions, classifications, objectives, constraints, network topology, decision variables and the solution methods. Ibarra-Rojas, Delgado, Giesen, and Munoz (2015) gave review on transit network planning problems and real-time control strategies suitable to bus transport systems.

Problem and Definition of UTNDP
A good understanding of some common terms is essential for modelling the UTNDP. These include: (1) the problem objectives; (2) problem parameters such as the decision variables, the structure of the road network, patterns and characteristics of the passengers travel demand, strategies of operation that might enhance the transit system's capacity and performance, as well as the constraints of the problem; (3) solution techniques which could be exact or heuristics. These terms are discussed in detail in Kepaptsoglou & Karlaftis, (2009).

Farahani et al. (2013) outlined several definitions of UTNDP as commonly provided in the literature, and adopted the definition according to Magnanti & Wong (1984). The UTNDP concerns the entire hierarchy of transportation planning process, which include: operational, tactical and strategic decisions. Ceder and Wilson (1986) divided the UTNDP into five stages: network design, frequency setting, time table development, vehicle scheduling and driver scheduling. In an ideal scenario, all five stages are supposed to be optimised together for interaction and feedback, so as to achieve a high quality results. However, this is not feasible because each phase is NP-hard in its own right (Magnanti & Wong, 1984). Subsequently, Chakroborty (2003) subdivided the UTNDP into urban transit routing problem (UTRP) and urban transit scheduling problem (UTSP). This classification has been widely accepted as the most commonly used problem type of the UTNDP.

Formally, the transit system plan can be decomposed into transit network design, vehicle schedule plan, and drivers dispatching plan (Zhao et al., 2015). According to Fan and Machemehl (2006), in recent decades, a lot of attention have been paid to address the UTNDP and past approaches that were employed to tackle it and this can be divided into three classes: (1) practical guidelines and procedures; (2) mathematical (analytical) optimisation models for idealised situations; and (3) metaheuristics for more practical problems. The computational complexity of (1) and (2) cannot address the realistic road network size because the size of the variables is large (Guan, Yang, & Wirasinghe, 2006; Murray, 2003). The advancement of computing technology coupled with the combinatorial nature of UTNDP has enabled a number of researchers to introduce and utilise different heuristics and metaheuristic algorithms.

Kepaptsoglou and Karlaftis (2009) summarised the practical goals of UTNDP as:

a. Transit user benefit maximisation which includes minimisation of costs such as travel, access, waiting, and transfers; maximisation of service coverage, while system benefits are represented by maximum utilisation and quality of service.
b. Transit operator cost minimisation with fleet size minimisation, profit maximisation, route set length minimisation, vehicle operation hour's minimisation, and fuel consumption minimisation as proxies.
c. Total welfare maximisation which is represented by the minimisation of passenger and operator costs.
d. Capacity maximisation of individual transit route capacity and vehicle capacity.
e. Energy conservation- protection of the environment from emissions and noise.
f. Individual parameter optimisation such as maximum allowable number of transit routes, transit route lengths, as well as the load factor.

In practice, some of the real-life constraints of UTNDP can be briefly summarised as follows (see, Baaj & Mahmassani, 1995; Pattnaik, Mohan, & Tom, 1998; Tom & Mohan, 2003; Mauttone & Urquhart, 2009; and Yan, Liu, Meng, & Jiang, 2013):

a. Demand covering – measures the percentage of passengers who travel directly or indirectly from the origin to the destination with at most two transfers.
b. Route length – implies that the length of the transit route should not be greater than a maximum allowable value because of difficulty in schedule maintenance. Likewise, the route length should not be lower than a minimum value to ensure connectivity or service quality.
c. Maximum number of routes – in consideration of the operators' resources and the desire to maximise profit.
d. Frequency – requires that the service frequency on the resulted transit route should be bounded, exceeding the maximum operationally implementable value, schedule maintenance become difficult. Similarly, it is impossible to provide a very low service frequency in an urban setting, it will be understood by passengers as absence of service.
e. Load factor – reflects the tolerance for the number of standing passengers.
f. Fleet size – imposes additional condition for the frequencies and operators resources.
g. Travel time reliability – a parameter that reflect the degree of service stability offered by the transit system.

Urban transit routing problem

The UTRP which is the first phase of the transportation planning process involves the construction of transit routes on a given road network with corresponding street segments travel times and predefined demand (stop) points base on the objectives associated constraints defined by the stakeholder(s). UTRP is considered as one of the most strategic planning phase in the UTNDP (Ceder & Wilson, 1986). The fact is, the structure of route design will have considerable impact on frequency setting, vehicle and crew scheduling. Besides, operators have the least flexibility in altering the routes. The route network is a subgraph of the transportation network, that is, from each station it is possible to travel to every other station by utilising route sets of the transportation network. Most of the studies in the literature attempted to optimise the transit user cost, operator cost, or both for tackling the UTRP. The proxies for transit user cost are total travel time (i.e. waiting, in-vehicle and transfer penalty), average travel time, and goodness of route sets (fitness). Operator cost is designated by total route set length which indirectly accounts for the vehicle kilometer (mileage), vehicle operation hours, fuel consumption and driver schedule.

For UTRP, the most common decision variables are the routes which satisfy the following constraints (Mumford, 2013; Chew, Lee, & Seow, 2013):

a. Cycles and backtracks are not allowed in a route.
b. Length of route which is commonly measured in number of nodes or time must be bounded within predefined minimum and maximum.

c. In a route network, the routes are connected to each other to form a complete route set.
d. Route network is composed of finite number of routes, set by the transit operator.
e. Route network is connected, but does not mean all demand is necessarily satisfied.
f. Demand, travel time, and distance matrices are symmetrical along the same route.
g. Demand level is inelastic throughout the period of study and shortest travel time guides passenger choice of routes
h. Policy headway is not considered, but it is assumed that there is adequate vehicles and capacity, and overall travel time consist solely of in-vehicle travel time and penalties for passenger transfer.

Urban transit scheduling problem
The UTSP is concerned with the development of schedules for the arrivals and departures of vehicles at the demand points (stops) of the given route network for a given number of available vehicles. A transit schedule is considered efficient if it minimises transit user's waiting time while operating within a set of resource and service related constraints.

For UTSP, the most common problem variables include the routes, departure time, and arrival time satisfying the resource and service related constraints (Chakroborty, 2003):
a. Fleet size: the number of vehicles (limited and fixed) available for plying particular route.
b. Vehicle capacity: the capacity of each vehicle is finite.
c. Stopping time bounds: a vehicle is not allowed to wait excessively at the demand point.
d. Policy headway: it is required to maintain a minimum service frequency on a given route.
e. Maximum transfer time: a transit user on the network should not wait too long at a transfer point.

Population-based Metaheuristics for UTNDP
The voluminous amount of research in the field of UTNDP can be categorised into: (1) models that tackle UTRP or UTSP exclusively, and (2) models that address UTRP and UTSP sequentially or simultaneously. In this paper, four well-known population-based metaheuristics, namely, genetic algorithm (GA), particle swarm optimisation (PSO), bee colony optimisation (BCO), and ant colony optimisation (ACO), that are very popular in tackling the UTNDP will be thoroughly reviewed.

Lately, the limitations associated with individual metaheuristics has enabled researchers to develop hybrid metaheuristics in order to improve the efficiency and flexibility that may result when tackling large-scale and realistic problems (Blum, Aguilera, Roli, & Sampels, 2008). A hybrid metaheuristic is an approach that involves skilful combination of a meta-heuristic with another optimisation technique. Consequently, several hybrid metaheuristics have been used for solving the UTNDP (Zhao & Zeng, 2006; Liu, Olszewski, & Goh, 2010; Szeto & Wu, 2011).

Genetic Algorithm
Genetic algorithm (GA) is a probabilistic search technique which uses the mechanics of the survival of the fittest and natural genetics (Holland, 1975). GA is initialised with a set of individual (called population), and each individual represents a solution that is feasible to the given problem. The individuals' fitness in the population is determined for each iteration (generation in GA terminology). Fitter individuals have a better chance of being selected from the population through a mechanism of selection to produce offspring for the next generation through a reproduction process (crossover and mutation). The selection of fitter individuals is performed for many generations with the hope of obtaining a population that is largely better than the original in terms of fitness.

GAs for UTRP

Pattnaik et al. (1998) presented a GA that involves two stages for addressing the UTRP. At the initial stage, a population of feasible route sets is generated, then the best route set is determined by GA. The chromosomes are coded into fixed and variable binary strings. The size of route the route sets are specified in advance in the case of the fixed string length scheme such that from the candidate route set the best one is determined. For the variable string length scheme, the best route set is determined by iteratively varying the route set size thereby selecting the size of the route set and the route sets simultaneously; however this demands complex coding. A part of real-life network is used to test the model. If computation time is not a constraint, the fixed string length model is found to be slightly superior to the variable string length model in terms of the other descriptors considered to compare the effectiveness of the two models.

Chien, Yang, & Hou (2001) developed a GA model for optimising the location of bus route and its frequency based on the irregular grid street patterns, intersection delays, realistically geographic variations, and different demand distributions. The aim is to optimise the total cost. The result indicates that for real-life urban networks it may be intractable computationally because if the number of links (streets) is increased it will lead to drastic increase in the population of bus routes that are feasible. An exhaustive search algorithm is utilised to validate the optimal solution. Hence the proposed GA converges efficiently to the optimal solution.

Bielli, Caramia, and Carotenuto, (2002) presented a novel approach for evaluating the fitness function in a GA for optimising bus network. GA is applied iteratively to create a new population, where every individual member of the population is evaluated based on some performance indicators for the studied networks by analysing the allocation of the corresponding origin-destination demand. The focus is to develop an algorithm that determines the best network in terms of demand and transport satisfaction. An experimental study is conducted on a realistic transport network. Computational results obtained demonstrate initial improvement on the value of initial fitness of about 90%.

Chakroborty & Wivedi (2002) introduced a GA based optimisation procedure such that initial candidate route sets are generated on an existing road network and travel demand data. The GA then evolves "optimal" or "efficient" transportation route networks. Computational results outperform the existing procedures by several authors on Mandl's Swiss network.

Chew et al. (2013) solved a bi-objective UTRP utilising GA to optimise passengers' and operators' costs. A set of parameters commonly used by other authors in the literature is adopted to evaluate the quality of the route set generated. A repair mechanism called an adding-node procedure to transform an infeasible solution to a feasible one is employed in the proposed algorithm. The GA operators include route crossover and identical-point mutation which have been considered simple but effective. The algorithm is executed by switching the objective function after the first objective has converged. The benchmark Mandl's Swiss network is used to evaluate the proposed GA, and in most cases the result obtained outperforms the previous best published results.

Miandoabchi, Daneshzand, Szeto, & Farahani (2013) addressed the problem based on multi-objective optimisation such that for a given initial network consisting of two-way links, candidate lane addition, and link construction projects. Several objectives are optimised including (1) the combination of one-way and two-way links, (2) the selection of network capacity projects, and (3) lane allocations on two-way links in order to optimise the network reserve capacity and performance indicators associated with two new travel times.

Nayeem, Rahman, & Rahman (2014) proposed two models based on GA (GA with elitism and GA with increasing population) with inelastic demand to tackle the UTRP with the objectives to maximise the number of satisfied passengers, minimise the overall transfers, and minimise the overall travel time of all

served passengers. GA with elitism is found to be competitive with the literature. However, the GA model with increasing population outperforms all published results.

GAs for UTSP

Chackroborty, Deb, & Subrahmanyam (1995) presented a GA model to solve a transit scheduling problem based on a mathematical programming framework in which the total transfer time plus the initial waiting time of passengers at the origin station is minimised. The experimental results from several test problems show that the proposed algorithm is able to generate optimal schedules within acceptable CPU time.

Shrivastava & Dhingira (2002) proposed a model for coordinating schedules between public buses and suburban trains using GA. A schedule optimisation model is used to determine the schedule coordination on an existing feeder routes that were constructed for both bus and train stations. The model attempts to optimise transfer time between the suburban trains and the public buses, as well as the operating costs of the public buses. The constraints include the transfer time between the arrival of a train and the departure of connecting buses does not exceed a maximum value, minimum time available for transfer, load factor bounds, and demand satisfaction. The study is conducted in Mumbai and the results indicated that the demand of buses on feeder routes developed is better than the existing feeder routes.

Kidwai, Marwah, Deb, & Karim (2005) presented a bi-level optimisation model for bus scheduling. For each individual route base on the load feasibility, the minimum bus frequency is determined at the first level. Assuming that the size of fleet obtained at first level is the upper bound, GA is then used to minimise the fleet size obtained at first level but in this stage all the routes are considered together. A road network of Burdwan city in India is considered for the proposed model.

Yu, Yang, & Yao (2009) proposed a two-level optimisation model for designing the frequency of companies' buses. The aim is to optimise passenger cost expressed in total travel time, while the constraint is company's total fleet size that takes into account the passenger route choice behaviour. An optimal criterion for assigning transit trips to the bus route network is considered as lower level objective. The bi-level model is tackled iteratively by the combination of GA and a label-marking algorithm. Computational result demonstrates that the local level of service of one of the companies is improved as well as several companies when integrated properly.

Mazloumi et al. (2012) developed two stochastic optimisation algorithms for determining transit schedules for a fixed bus route. They define the transit schedule design as an optimisation problem to determine the schedule of transit with location of timing points as the problem variables together with the slack time value corresponding to each timing point. A micro-simulation is performed to generate a transit route in Melbourne, Australia on which the algorithms were tested. Two parameters are used to compare the algorithms: effectiveness and computational efficiency in obtaining the optimal cost. The ACO outperforms the GA in terms of efficiency whereby less schedule designs are evaluated to achieve an optimal solution.

Afandizadeh, Khaksar, & Kalantari (2013) presented a bus fleet optimisation model based on three step procedures: (1) the design of network, (2) the determination and assignment of frequency, and (3) the evaluation of the network. GA was utilised to solve the problem, including bus assignment at depots. The model is calibrated on the benchmark Mandl's Swiss network and numerical results are competitive with several previous models. The model is later used to design Mashhad bus network in Iran.

GAs for UTNDP

Gundaliya, Shrivastava, & Dhingra (2000) developed a GA for solving routing and scheduling simultaneously. Routes and associated frequencies are encoded using a single binary string with the required tolerance. The objective is to minimise the sum of passengers' and operator's costs. The proxy for passenger cost is the sum of waiting time, in-vehicle travel time, and transfer time whereas operator cost is

vehicle operating cost of the buses. The constraints include the feasibility of the load factor, fleet size, and overloading of links. The benchmark Mandl's Swiss network is used to test the model and the result gives better-optimised values over other existing results in the literature.

Tom & Mohan (2003) developed a GA model that tackles the UTRP and UTSP simultaneously (i.e. UTNDP) in which the total system cost per trip is minimised. The route details is coded using binary coding scheme and the route frequency is incorporated as a variable at the same time. The model is tested on a medium-sized network of Chennai, India. Computational results indicated that the model yields the minimum waiting and travel time compared to the fixed string length model, but less efficient compared to the variable string length model.

Ngamchai & Lovell (2003) proposed a model that uses GA to optimise bus route network design, where for each route a distinct frequency of service is incorporated. Depending on the pattern of demand and the configuration of the network, the GA uses integer representation together with seven new evolutionary operators to search the solution explicitly. The proposed GA model proved to be better than the binary coded GA in terms of efficiency.

Chakroborty (2003) identified the sources of complexity associated with UTNDP which make it computationally intractable using traditional methods. He proposed a GA to solve the problem. Computational results demonstrated that the techniques based on the GA have recorded significant success in tackling transportation network design problem. The author concentrated on demonstrating the effectiveness of GA for addressing the UTNDP, identification of the characteristics of the UTNDP that poses difficulty for traditional methods, and finally direction suggested for developing techniques that are based on a GA for tackling problems that are related to the UTNDP.

Fan & Machemehl (2006) presented a GA-based model in a systematic way of exploring the features associated with optimal UTNDP through a multiobjective mixed integer nonlinear formulation. An experimental network is calibrated to obtain the computational results together with post-optimality analyses. Some of the unique characteristics related to this methodology include: (1) the solution approach applies hybrid transit trip demand assignment models and incorporates corresponding paths that involve long-walk explicitly and features of transfer between routes considering both more general and practical scenario; (2) the technique assumes that the transit demand is elastic and employs iterative approach to compute simultaneously the transit demand, optimal route set, and the frequencies of the routes; and (3) finally, fine-tuning some parameters such as route network size, demand aggregation, and redesign of existing route networks to observe clearly their impact for the UTNDP.

Zhao & Zeng (2006) proposed a Simulated Annealing-Genetic Algorithm (SAGA) for transportation network optimisation. The aim is to develop an effective algorithm tool for the optimisation of a large-scale transportation route network to optimise two criteria: minimising transfers with reasonable route directness and maximising demand coverage. A three step procedure is utilised including: transit route representation, establishment of the constraints of the network, and the stochastic search scheme. A computer program is developed to implement the methodology by using it on previous literature, and onto a real-life large-scale transit network optimisation.

Beltran, Carrese, Cipriani, & Petrelli (2009) introduced a method for solving the UTNDP with variable transit demand and multimodal framework, giving attention to the effects of emissions from vehicles in an urban setting. The aim is to optimise passengers, operator, and external costs. The methodology consists of two stages: an initial route generation heuristic and a GA to determine a sub-optimal route sets and corresponding frequencies simultaneously. Extensive experiments have been performed on a realistic network to determine the compromise solution between the externalities associated with traditional and green lines.

Liu et al. (2010) proposed a hybrid strategy to tackle bus network design and frequency setting simultaneously on existing road network with inelastic transit demand and bus stations. The methodology involves first, candidate route sets are generated, then a combined SA and GA approach is employed to select the optimal route set. The SA optimises the passenger and operator costs, while new solutions are generated by the GA. Passenger assignment restricted to two paths is conducted during solution evaluation. The method is tested on a benchmark network and four theoretical grid networks of different dimension. A number of GA parameters are utilised and tested. The results indicate that the method can experience a scalability problem.

Szeto & Wu (2011) developed a GA model to solve a trunk bus network design problem of a residential area in Hong Kong. The objective is to minimise passenger cost and the number of transfer using the existing bus transit services. The methodology include hybridisation of GA that handles problem of route design with a local search heuristic that tackles the problem of frequency setting. A novel scheme of chromosome representation and evolutionary parameters are proposed to explore the solution space. To avoid the premature convergence, a diversity control mechanism is introduced in the methodology on account of a new definition of hamming distance.

Arbex & da Cunha (2015) proposed an efficient GA to solve the UTNDP by changing cyclically the user and operator costs to be optimised along the generation based on multiobjective nature of UTNDP. The proxys for passengers' cost are waiting time, in-vehicle time, and overall transfers whereas the fleet size required to operate the route network represents the operators' cost. To avoid loss of feasibility of mutated and newly generated individuals, the GA applies local search procedures. Numerical experiments are conducted on the benchmark Mandl's Swiss network and instances with different network data to evaluate the pareto optimal solutions.

Most recently, Zhao et al. (2015) introduced Memetic algorithm (MA) to optimise the UTNDP. The objective is to minimise passenger cost and dissatisfied passenger demand. For computational efficiency of the model, the MA is embedded with local search parameters including relocation move, swap move, and 2-opt move (type A and type B) based on the traditional GA. The model has been calibrated on benchmark networks reported in the relevant literatures.

Most literature adopted the generalized user cost, operator cost or both as the objective function. However, the pollution (emission), energy consumption, or both have not been explicitly reflected in the formulation of the objective function. In addition, no work has been found that considered the assumption of heterogeneous fleet. Despite the ability of GA in solving diverse and related UTNDP models, there is no study of GA to large multimodal transit networks, in which the passenger assignment algorithm should model multimodal passenger choices. Similarly, there is no study of GA in the literature that addresses problem scenarios with longer planning horizons with temporarily elastic demand, as well as stochastic in-vehicle travel time. Some hybrid GAs have been developed for the UTNDP but the hybridization with other population based metaheuristics such as PSO, Differential Evolution (DE), and Chemical Reaction Optimisation (CRO) are yet to be explored. Additionally, only Agrawal and Mathew (2004) have devoted attention to parallel computing strategies using GA to improve the computational efficiency.

Particle swarm optimisation
Particle Swarm Optimisation (PSO) is a swarm intelligence-based optimisation technique inspired from observing the behaviour of some natural group organisms such as fishes and birds swarm. First introduced by Eberhart & Kennedy (1995), the PSO operates by creating a swarm (population) of particles representing feasible candidate solutions. A simple model that describe the movement of these particles in the search space is established in terms of velocity, personal best, and global best. The particles adjust their movements based on their own best known experience in the search space as well as the best known of the

entire swarm. As a result of these movements, better positions are more likely to be discovered, and these will serve as a guide to the movements of the entire swarm. The algorithm terminates when an optimal solution is found.

PSO for UTRP
Babazadeh, Poorzahedy, & Nikoosokhan (2011) formulated a bi-level optimisation model employing a PSO for the road network design problem. Application of the algorithm on the Sioux Falls test network indicates that the performance of PSO approach is comparable with hybridised ACO.

Bagherian, Massah, and Kermanshahi (2013) proposed a discrete PSO to solve the UTRP through mixed integer formulation. The methodology consists of two stages. A heuristic for generating feasible route sets is initially developed, then a DPSO is employed to determine the optimal route sets. The coverage index is optimised subject to the operator cost which is the upper level constraints. The solution is utilised for solving the problem in different level of budget. In all the scenarios, the optimum solution is found upon a plausible number of evaluations. The results confirmed the capability of the DPSO with a significant decrease in the total computational cost.

Recently, Kechagiopoulos & Beligiannis (2014) presented a PSO for solving the UTRP with emphasis on proper representation of particles and evaluation procedure. The methodology considers the level of service offered to each transit user as well as the operator cost. Experimental results on the benchmark Mandl's Swiss network are compared with other published results indicate that the proposed PSO is competitive.

Based on the literature, the research mostly concentrated on the application of GA for UTNDP with only a few studies based on PSO for UTRP. For instance, there is no research been carried out on PSO for UTSP, and UTNDP. Therefore, further research to validate the solution quality and efficiency of PSO can be conducted. Similar to the GA, the research gaps for PSO include: study of hybrid PSO, parallel PSO, objective functions (in terms of pollution and energy consumption), and assumptions of heterogeneous fleet, variable in-vehicle travel time, and elastic travel demand in UTNDP. In addition, real-life transit network and multi-objective type PSO is not given significant attention by the literature.

Bee Colony Optimisation
Bee colony optimisation (BCO) is first introduced by Karaboga (2005). This metaheuristic mimics the behaviour of honey bees in food foraging. The employed and scout bees constitute the colony in this algorithm. Based on their memories, the employed bees search for food in the vicinity of the food source, while the scout bees search for a new source of food. In every situation both group of bees share their information with other bees within the hive (Basu, 2011). Two actions are utilised by the bees which include forward pass where individual bee explores the search space, and in backward pass information is shared with other bees. Gradually, artificial bees collectively construct and/or improve their solutions. The BCO algorithm is executed in iterations until some termination criteria is reached.

BCO for UTRP
Nikolić & Teodorović (2013) proposed a BCO algorithm for addressing the UTRP. The aim is to optimise in terms of travel demand, the number of transit users satisfied, overall passenger transfers, and the user cost of all served transit users. The methodology initially generates candidate solutions using a simple greedy algorithm, then improvement version of BCO is employed to determine the best set of routes. Computational experiments on the benchmark Mandl's Swiss network confirms that the BCO metaheuristic is effective compared to most of the approaches in the literature.

BCO for UTNDP
Szeto & Jiang (2012) proposed a model for solving UTNDP where the passenger transfers and the overall travel time of users is optimised through a hybrid enhanced artificial bee colony (HEABC) algorithm. The

HEABC algorithm determines the route sets, while the frequency assignment is tackled by a neighbourhood search heuristic which allows the exploration of various potential routes. In terms of the parameters including overall travel time, overall fuel cost, maximum intermediate stations, number of transfers, maximum headway; the results obtained by HEABC outperforms hybrid GA and a variant of the HEABC. Additionally, the HEABC is found to produce a network design better than the existing one in terms of the descriptors mentioned above. However, no attention is given to the comparison of the study with other algorithms in the literature.

Nikolić & Teodorović (2014) recently developed an efficient BCO which addresses the UTNDP by determining the segments to be inserted in the construction of the route network, combine chosen segments into transit routes, and the passenger assignment approach provided in the papers by Shih & Mahmassani (1994) and Baaj & Mahmassani (1995) is adopted to determine vehicle frequency simultaneously. Computational experiments on benchmark problems yield results that outperform some approaches in the existing literature.

Szeto and Jiang (2014) proposed a bi-level UTNDP where transit network design and frequency settings are solved together. A mixed integer non-linear programming formulation that optimises number of passenger transfers is utilised at the upper level, while the passenger assignment task with capacity constraints is handled at the lower level. The transit route structures are determined by a hybrid artificial bee colony, whereas the capacity-constrained passenger assignment model is tackled using Simplex method. A repair mechanism for the frequency assignment as well as lower bound screening to improve efficiency is developed. In addition, sensitivity analysis is conducted on the objective function value and likely solutions from a number of designs. A comparison of the proposed algorithm with a GA in the Winnipeg scenario is performed to confirm its superiority.

Based on the literature, as mentioned above, the majority of the metaheuristics proposed for the problem employed GA with different coding schemes at the expense of more recent population-based metaheuristics such as BCO. In particular, no research on BCO for UTSP is found. Similar to GA and PSO metaheuristics, the research gaps are: hybridisation of BCO by incorporating the parameters and strength of classical metaheuristics, implementation of parallel BCO, objective function expressed in terms of environmental factors, the assumption of non-uniform vehicle capacity, and BCO tested on real-life transit networks of the UTNDP.

Ant Colony Optimisation
The ant colony optimisation (ACO) is first proposed by Dorigo, Maniezzo, & Colorni (1991). It was inspired by the behaviour of ants in finding paths from the nest to food. In a natural sense, ants (initially) move randomly, and return to their colony having found the food by discharging chemical substance called pheromone. The pheromone trails enable the other ants to avoid travelling at random, but rather follow the trail while returning and at the same time reinforcing it once they found the food. As a result, when an ant finds a good path from the nest to a food source, other ants are likely to follow that path, and positive feedback finally enables all the ants to follow a single path. The idea of the basic ACO is to mimic this behaviour with 'artificial ants' travelling around the network which constitutes the problem to be tackled.

ACO for UTRP
Yang, Yu, & Cheng (2007) developed a parallel ACO for bus network optimisation. The objective to be maximised is the direct traveller density based on travel demand of the whole bus network. The direct traveller density according to the authors is the number of passengers travelling without transfer per unit length. A simple network (17 nodes and 29 links) is used to validate the model and the ACO, while for larger instance the data in Dalian City (2300 nodes and 3200 links), China, is used. The obtained results indicate that the travel time and number of transfers has decreased significantly from the generated bus network.

Mohaymany & Gholami (2010) presented an ACO for a multimodal feeder network design problem. The objective is to propose algorithm that addresses at least two modes in the network design where the passenger, operator and social costs are optimised. The variables include route sets, route frequency, and stations configured to specific mode. The methodology consists of first designating the terminals, and then a route set is constructed from each terminal to one station. Then for all design modes, the frequency on each route is calculated. For each route the best mode is selected, and finally the total cost of the multimodal network is established. The ACO algorithm is applied until the stopping criteria is reached. A theoretical network is first designed with only one bus network in order to make comparison with performance measures of multimodal networks with seven different scenarios. Results indicate that multimodal networks provides better passenger costs, but difference exists for operator costs base on costs per unit where in some situation operator costs are lower and, in other cases, they are higher than those of unimodal networks.

ACO for UTSP
Yu, Yang, Cheng, & Liu (2005) developed an optimisation model that considered the zonal stations (origin-destinations) and the road network to address the bus network design problem. The objective is to optimise transit user transfers and flow of passengers per unit length subject to the constraints of line length and non-linear rate. The methodology involves the incorporation of a heuristic pheromone distribution criterion into coarse-grain parallel ACO, so that base on the objective value the ants' can explore the solution space effectively, while the parallel ACO handles the computational efficiency of the algorithm. The model is calibrated on survey data of Dalian city and the results yield bus network with minimum user cost and number of transfers.

Poorzahedy & Rouhani (2007) presented a hybrid approach using ant system to handle the network design problem. The methodology consist of attempts to use the ant system metaheuristic only to address the network design phase and hybridise the ant system with GA, tabu search, and SA algorithms. Hybrid models have been developed and calibrated on a realistic Sioux Falls network. Results indicate that the hybrid models outperform the sole ant system in terms of effectiveness.

In contrast to BCO, efforts to develop ACO for UTNDP, as well as to tackle the transit network design and frequency setting problem (TNDFSP) simultaneously deserves some attentions. Similar to the metaheuristics mentioned above, the research gaps for ACO include: hybridisation of ACO by incorporating the parameters and strength of classical metaheuristics, objective function that is different from the conventional welfare maximization, and the constraint of non-uniform vehicle capacity to address the UTNDP.

Conclusion
In this paper, we have presented a comprehensive review of the UTNDP by focusing on the population-based metaheuristic approaches. Based on the problem domain, the UTNDP is categorised into UTRP, UTSP, and UTNDP with corresponding objectives, solution methods used, and application network studied. In addition, we highlighted some research gaps in the literature, which require more studies for the four metaheuristics considered.

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